Spring 2026: Pseudodifferential Operators

HOMEWORK 5

Due: 2026-05-28

1. Recall Theorem 2.1.A of Taylor: Let $p(x,\xi) \in C^s_* S^m_{1,1}$, $p \in (1,\infty)$, and $0 < r < s$,. Then we have the bounds

$$P(x,D) : W^{r+m,p} \to W^{r,p}, \qquad P(x,D) : C^{r+m}_* \to C^r_*.$$

Use this to prove the following generalization (fill in and check the details of the outline provided in Taylor):

Let $p(x,\xi) \in C^s S^m_{1,\delta}$ with $\delta \in (0,1)$, $p \in (1,\infty)$, and $-(1-\delta)s < r < s$,. Then we have the same bounds.

2. Check and prove the elliptic regularity result (2.2.15) in Taylor: If $a(x,\xi) \in C^s S^m_{cl}$ is elliptic and $s > 0, \epsilon > 0$, then

$$u \in W^{m-s+\epsilon,p} \text{ and } Au \in W^{s,p} \Rightarrow W^{m+s,p}.$$

(Actually, this is not (2.2.15), which I suspect has a typo. Please me know if you think the original is correct.)

3. Prove Proposition B.1.C. in Taylor: If $p(x,\xi) \in S^0_{1,\delta}$ with $\delta \in [0,1)$, $p \in (1,\infty)$, and $s > n/p$, then

$$\|Pu\|_{L^\infty} \lesssim \|u\|_{C^0_*} \cdot (1 + \log(\|u\|_{W^{s,p}}/\|u\|_{C^0_*}).$$