Spring 2026: Pseudodifferential Operators

HOMEWORK 3

Due: 2026-04-30

1. Fill in the details of Proposition 0.3.A of Taylor (the asymptotic expansion of $a(x, y, \xi)$). Precisely,

a) Show that the full proposition reduces to the same proposition with the additional assumption that

$$\text{supp } a(x, y, \xi) \subseteq \{|x - y| \leq 1\}.$$

b) Adjust the proof to remove the $\langle \xi \rangle^n$ loss in equation (0.3.12) at the end of the proof.

2. Provide the proof of Proposition 0.3.C of Taylor (composition of pseudodifferential operators), except with the weakened assumption $0 \leq \delta_2 < \rho_1 \leq 1$.

3. The operator $P(x, D)$ is not self-adjoint in general, even if its symbol $p(x, \xi)$ is real. This is one motivation to consider an alternative correspondence between symbols and operators. The *Weyl quantization* associates to a symbol $p(x, \xi)$ the operator

$$(P^w(x, D)u)(x) = \int e^{i(x-y)\xi} p\left(\frac{x+y}{2}, \xi\right) u(y)\, dy d\xi.$$

a) Given $q(x, \xi) \in S^m_{\rho,\delta}$, $0 \leq \delta < \rho \leq 1$, produce a formula for $p(x, \xi)$ such that $P^w(x, D) = Q(x, D)$.

b) Find and prove an asymptotic expansion for $p(x, \xi)$ produced in part a), in terms of $q(x, \xi)$.